Enrollment No: _____________________ **Exam Seat No:**_____________________

C. U. SHAH UNIVERSITY

Winter Examination-2022

Subject Name :Complex Analysis

Subject Code :4SC05COA1 **Branch: B.Sc. (Mathematics)**

Semester: 5 **Date: 22/11/2022** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1		Attempt the following questions:	(14)
	a)	Define bilinear transformation.	01
	b)	State fundamental theorem of algebra.	01
	c)	State ML inequality.	01
	d)	Define harmonic unction.	01
	e)	Show that $u(x, y) = 2x - x^3 + 3xy^2$ is harmonic function.	02
	f)	Is $w = e^{\bar{z}}$ is entire? Justify.	02
	g)	Check whether $f(z) = 2x + ixy^2$ is analytic function ornot at any point.	02
	h)	Find invariant points of $w = \frac{z-1}{z+1}$.	02
	i)	Find arc length of the curve $z(t) = t + it, t \in [-1,1]$.	02

Attempt any four questions from Q-2 to Q-8

Q-2		Attempt all questions	(14)
	A	State and prove necessary condition for function to be differentiable.	07
	B	Show that $u(x, y) = e^{-2xy} sin(x^2 - y^2)$ is harmonic.	04
	C	Check whether $\lim_{z\to 0} \frac{\bar{z}}{z}$ exists or not? If it exits, find its limits.	03
Q-3		Attempt all questions	(14)
	A	Derive Cauchy Riemann equation in polar form.	06
	B	Find harmonic conjugate of $u(x, y) = y^3 - 3x^2y$.	05
	C	Prove that $f(z) = \bar{z}$ is nowhere differentiable.	03
Q-4		Attempt all questions	(14)
	A	Prove that if $f(z)$ and $\overline{f(z)}$ are both analytic in a domain D, then $f(z)$ must be constant throughout D.	05
	B	Find the analytic function $f(z) = u + iv$ if $u - v = (x - y)(x^2 +$ *4xy+y2.*	05
	C	Evaluate $\int_C \frac{z+2}{z} dz$, where C is the circle $z = 2e^{i\theta}$ $(0 \le \theta \le 2\pi)$.	04

Q-5		Attempt all questions	(14)
	A	Find $\int_C \pi \exp(\pi\bar{z})\, dz$, where C is the boundary of the square with vertices at the points 0,1,1+i and i, the orientation of C being in the positive direction.	05
	B	Show that if C is the boundary of the triangle with vertices at the points 0, 3i and -4, oriented in the counterclockwise direction (starting from 0), then $\left|\int_C (e^z - \bar{z})dz\right| \leq 60$.	05
	C	Find the value of integral $\int_C z^2\, dz$ where C is contour which is a part of $y = x^2$ from $z = 0$ to $z = 1 + i$.	04
Q-6		Attempt all questions	(14)
	A	State and prove Morera's theorem.	05
	B	Evaluate $\int_C \bar{z}\, dz$, where C is the line segment from $z = 1 - i$ to $z = 3 + 2i$.	05
	C	Evaluate $\int_C \frac{z}{9-z^2} dz$, where C is the positively oriented circle $|z| = 2$.	04
Q-7		Attempt all questions	(14)
	A	State and prove Liouville's theorem.	05
	B	Let C be the circle $|z| = 3$, described in the positive sense. Show that if $g(z) = \int_C \frac{2S^2-S-2}{S-z} ds \; (|z| \neq 3)$, then $g(2) = 8\pi i$.	05
	C	Evaluate $\int_C \frac{e^{2z}}{(1-z)^4} dz$, where $C: |z| = 2$.	04
Q-8		Attempt all questions	(14)
	A	Find the image of $|z + 1| = 1$ under the transformation $w = \frac{1}{z}$ and draw its rough sketch.	05
	B	Find mobious transformation that maps the points $z_1 = -1, z_2 = 0, z_3 = 1$ onto $w_1 = -i, w_2 = 1, w_3 = i$ respectively.	05
	C	Transform the curve $x^2 - y^2 = 4$ under the mapping $w = z^2$.	04